

Comparison of System Optimization Methods: LQR vs. LQT on the Output Response of IG-42CRGM DC Motor

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| Article Info | Abstract |
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| <p>Article history:</p> <p>Received 02 July, 2025 Revised 15 October, 2025 Accepted 01 November, 2025</p> | <p>DC motors are widely utilized in industrial applications for their reliability and efficiency. To optimize their performance, it is crucial to employ control systems supported by mathematical modeling to predict motor responses under varying conditions. This study investigates the first- and second-order models of DC motors and examines the impact of internal disturbances (noise) on system performance. The output responses of two DC motors, the 42BLFX02 and Maxon EC-I 40 (70W), are compared under both undisturbed and noisy conditions using simulations. The results reveal that the second-order model offers a more stable response and better aligns with the desired target compared to the first-order model. Furthermore, the application of the Linear Quadratic Regulator (LQR) control method significantly enhances the speed and accuracy of reaching the motor set point. However, when noise is introduced, the LQR method fails to maintain stability, and the motor's output starts to mirror the disturbance pattern. These findings highlight that while LQR is effective under ideal conditions, its performance diminishes when exposed to disturbances. Therefore, additional strategies are necessary to ensure stability and optimal performance in real-world conditions, particularly in environments with significant noise or disturbances.</p> <p>Keyword: Optimal control, Linear Quadratic Regulator, DC Motor</p> |
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1. Introduction

DC motors are widely used electronic components that can be found in various aspects of daily life. These electric motors function as electromagnetic devices that convert electrical energy into mechanical motion [1]. The basic working principle of a DC motor involves maintaining an

opposing direction between the magnetic fields of the rotor and stator, which generates motion due to the repelling force [2-4]. Energized current flowing through the armature coil creates a magnetic field in a specific direction around the coil, causing rotation [5].

Generally, DC motors slow down under load and do not maintain a constant speed. Their speed can be adjusted by varying the input voltage. For instance, when a load increases and the motor slows down, its speed can be restored by increasing the voltage supply. Therefore, a controller is essential to maintain stable motor speed despite load variations [6][7]. The controller's fundamental role is to compare the actual output of the system with the desired reference input, determine the error, and generate a control signal to minimize this error toward zero [8-10].

Among the control strategies available for improving DC motor performance, Linear Quadratic Regulator (LQR) and Linear Quadratic Tracker (LQT) are two prominent methods [11]. The LQR method is designed to help the motor output closely follow the desired setpoint while minimizing issues like overshoot and undershoot [12]. LQR also offers robustness, reliability, and static gain generation, making it suitable for large-scale systems with multiple inputs, where efficient and economical control of multiple outputs is required. The method optimally minimizes a given cost function, or performance index [13][14].

On the other hand, LQT is a linear control strategy aimed at ensuring the system output accurately tracks a specified reference trajectory. LQT employs a model-based approach using affine state-feedback, combining traditional state feedback with an additional feedforward control term that depends on the reference signal vector [16]. It is often used in systems that require optimized tracking performance.

2. Research Method

2.1. DC Motor Identification


At this stage, the specifications of the DC motor used in the research are identified. The DC motor being studied is a brushless type, specifically the Maxon EC-I 40 with a power rating of 70 Watts. Below is the datasheet provided for the IG-42CRGM DC motor, which serves as a reference for comparison in this study.

DC Carbon-brush motor

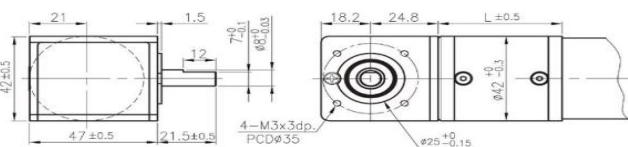
IG-42CRGM

01 & 02 TYPE

Note: The dimensions of the motor may vary slightly from what is shown on the datasheet. If the motor length dimension is ultra critical in your application please contact us (info@sdrobotics.com) for an accurate length before buying.



外形尺寸/ Appearance Size



unit:mm

齒輪馬達轉矩 & 速度/ Geared Motor Torque & Speed

| | 減速比 Reduction ratio | | | | | | | | | | | | | | | | | |
|------------------------------|------------------------|-----|-----|------|-----|-----|------|-----|-----|-----|------|------|------|-----|-----|------|--|--|
| | 1/4 | 1/4 | 1/3 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | | |
| 12V | 2 | 6.1 | 7.6 | 8 | 18 | 18 | 18 | 20 | 20 | 25 | 25 | 30 | 30 | 30 | 30 | 30 | | |
| 定額轉矩 Rated torque (Kg-cm) | 1400 | 405 | 325 | 248 | 120 | 98 | 76 | 63 | 45 | 31 | 24 | 13.5 | 10.9 | 9.5 | 8 | 6.5 | | |
| 定額轉數 Rated speed (rpm) | 1.6 | 5 | 6.2 | 8 | 15 | 18 | 18 | 20 | 20 | 25 | 25 | 30 | 30 | 30 | 30 | 30 | | |
| 24V | 1445 | 420 | 340 | 240 | 122 | 102 | 77.5 | 63 | 47 | 31 | 23.8 | 13.5 | 10.9 | 9.5 | 8 | 6.5 | | |
| 定額轉矩 Rated torque (Kg-cm) | 1.6 | 5 | 6.2 | 8 | 15 | 18 | 18 | 20 | 20 | 25 | 25 | 30 | 30 | 30 | 30 | 30 | | |
| 定額轉數 Rated speed (rpm) | CCW | | | | | | | | | | | | | | | | | |
| 軸心迴轉方向 Rotation direction | | | | | | | | | | | | | | | | | | |
| 長度 Length (L) | 32.5 | | | 39.2 | | | 45.9 | | | | | | 52.6 | | | 59.6 | | |

馬達單體型式/ Motor Data

| 定額電壓 Rated volt (V) | 定額轉矩 Rated torque (g-cm) | 定額轉數 Rated speed (rpm) | 定額電流 Rated current (mA) | 無負荷轉數 No load speed (rpm) | 無負荷電流 No load current (mA) | 定額輸出 Rated output (W) | 重量 Weight (g) |
|------------------------|-----------------------------|---------------------------|----------------------------|------------------------------|-------------------------------|--------------------------|------------------|
| 12 | 700 | 5700 | ≤ 5500 | 7000 | ≤ 900 | 41.3 | 360 |
| 24 | 570 | 5900 | ≤ 2100 | 7000 | ≤ 500 | 34.7 | 360 |

Figure 1. DC Motor Datasheet

From Figure 1, data can be taken regarding the specifications of the IG-42CRGM DC Motor which will be presented in Table 1.

Table 1. Specification DC Motor

| Model | Data | Units |
|-----------------|------|-------|
| Nominal voltage | 12 | V |
| Rated torque | 700 | g-cm |
| Rated speed | 5700 | Rpm |
| Rated current | 5500 | mA |
| No load speed | 7000 | Rpm |
| No load current | 900 | mA |
| Rated output | 41.3 | W |
| Weight | 360 | g |

From table 1 above, identification is then carried out regarding the DC motor used through mathematical calculations that require variable values obtained from the DC motor specification table data.

2.2. Mathematical model Order 1

General form of first order transfer function

$$G(s) = \frac{K}{\tau s + 1} \quad (1)$$

Based on the DC motor datasheet, the first order equation is obtained: Where $\tau = K.i$ so that

$$K = \frac{\tau}{i} = \frac{1,76}{5,5} = 0,314 \quad (2)$$

First order equation of DC motor:

$$G(s) = \frac{0,314}{1,76s + 1} \quad (3)$$

2.3. Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator (LQR) is a method widely applied in modern control theory. This technique relies on a state-space approach for system analysis [17]. Due to the simplicity and effectiveness of this approach, it is especially suitable for handling multi-input multi-output (MIMO) systems. The general form of a system using the state-space representation can be expressed as follows:

$$\dot{X} = AX + Bu \quad (4)$$

In principle, the LQR method searches for a control signal u that minimizes the performance index J .

$$J = \int (X^T Q_x + u^T R_u) dt \quad (5)$$

LQR finds the optimal control input law u^* . The constraints imposed by the Q and R matrices minimize the performance index. The closed-loop optimal control law is defined as:

$$u^* = -Kx \tag{6}$$

In this context, K represents the optimal feedback gain matrix. This matrix functions to minimize the performance index by determining the appropriate placement of the closed-loop poles. The value of K depends on the system matrices A and B, as well as the weighting matrices Q and R [18]. The feedback gain matrix K is derived by solving the Algebraic Riccati Equation (ARE). The solution to this equation yields P, which is a symmetric and positive definite matrix, and is defined as follows:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{7}$$

$$K = AX - BKx = (A - BK)x \tag{8}$$

By substituting equations (8) and (9), the expression becomes:

$$\dot{x} = AX - BKx = (A - BK)x \tag{9}$$

The block diagram illustrating the configuration of the LQR controller is presented in Figure 2.

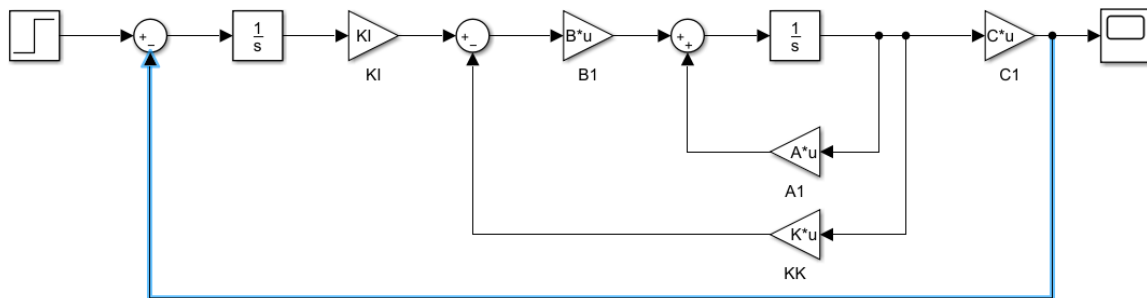


Figure 2. LQR Diagram Block

2.4. Linear Quadratic Tracker (LQT)

The Linear Quadratic Tracker (LQT) consists of the standard state feedback component from a linear dynamic system, combined with an additional feedforward control term [19]. This feedforward term is dependent on the reference signal vector, denoted as $r(t)$. The reference vector $r(t)$ is expressed as:

$$r(t) = [V_{ref}(t) \ 0]^T \tag{10}$$

Here, V_{ref} represents a reference voltage signal that varies over time. The LQT framework aims to minimize a quadratic performance index in order to generate optimal control decisions. This optimization can be mathematically formulated as shown in the following equation:

$$J = \frac{1}{2} \int_0^T [(x(t) - r(t))^T Q (x(t) - r(t)) + d(t)^T R d(t)] dt \tag{11}$$

In this formulation, Q and R are the state and control weighting matrices, respectively [20]. These matrices are selected such that $Q = Q^T \geq 0$ and $R = R^T > 0$. Due to the quadratic nature of the cost function, the control signal becomes proportional to the squared variation of the states. As a result, larger state deviations lead to stronger minimization efforts and, consequently, faster convergence rates. The optimal affine control decision is determined using the mathematical expression:

$$d(t) = -Kx(t) + K_{ff}v_{ref}(t) \tag{12}$$

Where

$$K = R^{-1}B^T P \tag{13}$$

$$K_{ff} = R^{-1}B^T((A - BK)^T)^{-1}H^T Q \tag{14}$$

The gain vector K is used to shift the poles of the system to synthesize an optimal controller. The optimal gain vector is derived from the positive definite symmetric matrix P . This matrix P can be obtained by solving the Algebraic Riccati Equation, as defined in the related formulation.

$$A^T P + PA - PBR^{-1}B^T P + H^T QH = 0 \tag{15}$$

2.5. System Block Diagram

A. Block Diagram of First Order DC Motor IG-42CRGM

The purpose of the first-order motor block diagram is to observe the original response of the IG-42CRGM DC motor when no control method is applied, as implemented in the Simulink simulation software.

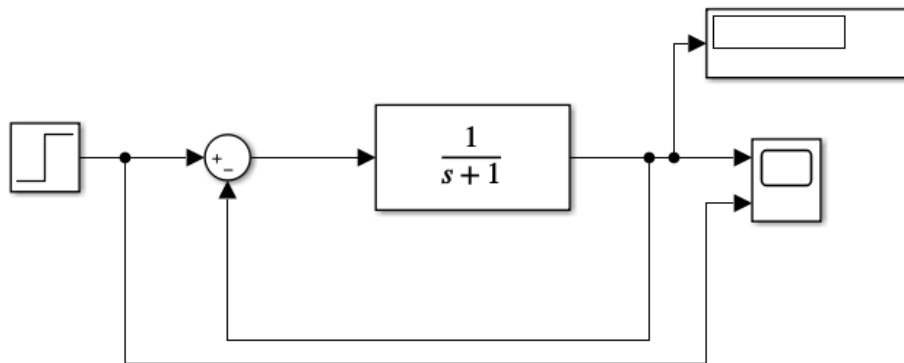


Figure 3. Block Diagram of First Order DC Motor

Figure 3 illustrates a first-order block diagram of a DC motor, which includes a single input and a single output. The input signal used is a step response. The transfer function within the diagram represents the first-order model of the DC motor. The resulting output response is displayed using a scope and display module to observe the maximum value achieved by the motor's response.

B. Block Diagram LQR DC Motor IG-42CRGM

The LQR block diagram for the IG-42CRGM DC motor is designed to analyze the motor's response when an LQR optimization method is applied. This implementation is carried out using Simulink software to evaluate how the control strategy affects the system's performance.

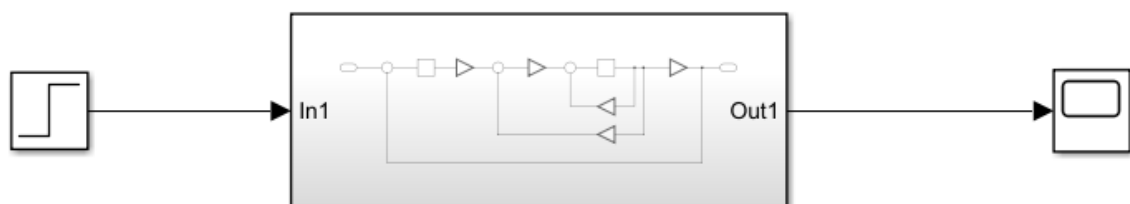


Figure 4. Block Diagram LQR DC Motor

C. Block Diagram of LQR DC Motor IG-42CRGM Watt with Noise

The LQR block diagram with noise for the IG-42CRGM DC motor is intended to observe the motor's response when both the LQR optimization method and external noise are applied to the system. This simulation is conducted using Simulink software to evaluate the controller's effectiveness under disturbance conditions.

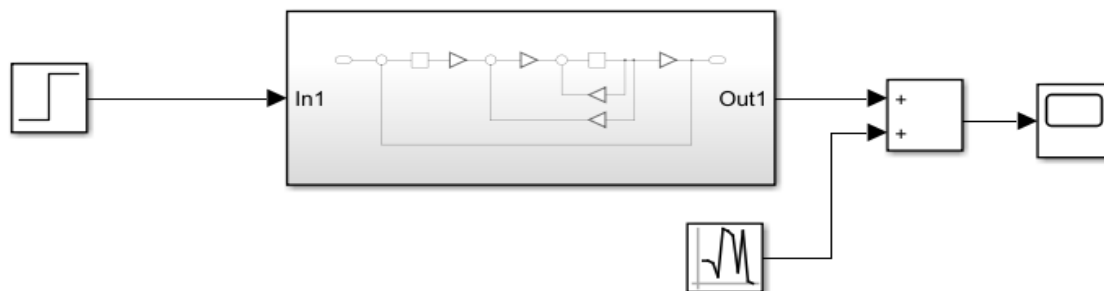


Figure 5. Block Diagram LQR DC Motor with noise

D. Block Diagram LQT DC Motor IG-42CRGM

The LQT block diagram of the IG-42CRGM DC motor aims to evaluate the motor's response when the LQT optimization method is applied. This process is conducted using Simulink software to determine how effectively the system follows the desired reference trajectory.

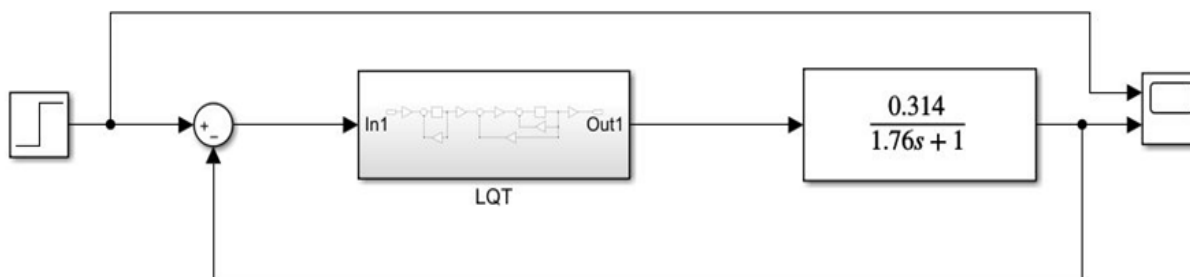


Figure 6. Block Diagram LQT DC Motor

E. Block Diagram of LQT DC Motor IG-42CRGM Watt with Noise

The LQT block diagram with noise for the IG-42CRGM DC motor is designed to analyze the motor's response when both the LQT optimization technique and noise disturbances are introduced into the system. The simulation, carried out in Simulink, helps assess the controller's performance under non-ideal operating conditions.

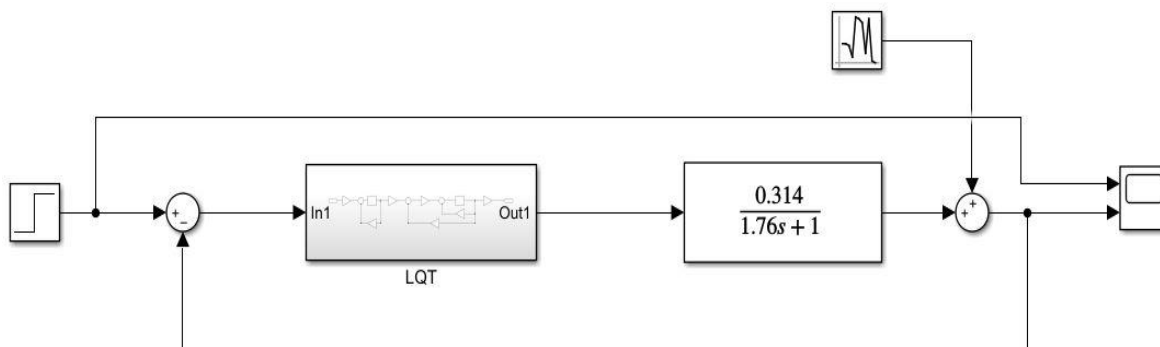
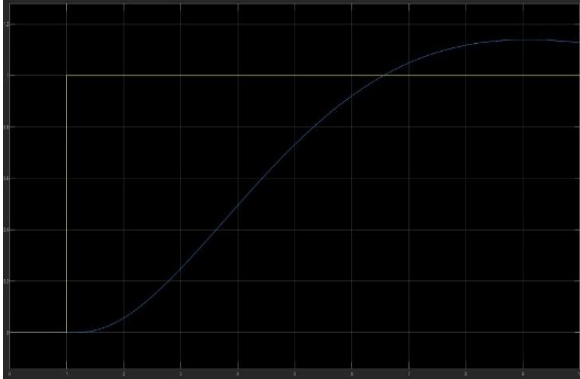
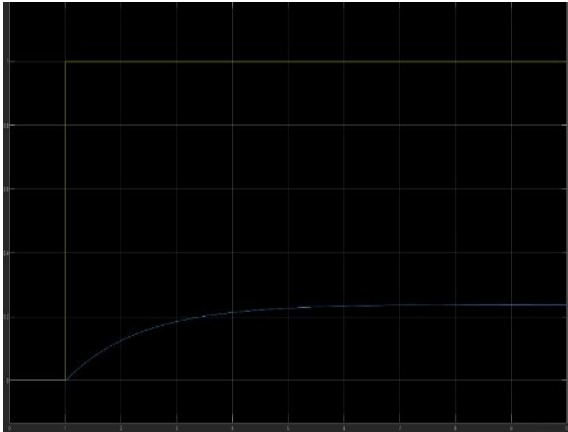


Figure 7. Block Diagram LQT DC Motor with noise

3. Results And Discussion

3.1. Simulation Result of DC Motor

Table 2. Simulation result LQR and LQT without noise


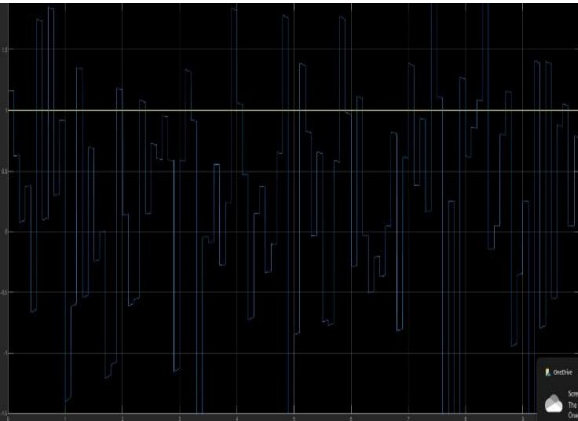
| Method | Scope | Without Noise | |
|--------|--|---------------|------------|
| | | Set Point | Undershoot |
| LQR |  | 1 | - |
| LQT |  | 1 | - |

Based on the simulation results, it can be concluded that the first-order DC motor system using the Linear Quadratic Regulator (LQR) optimization method successfully reaches the desired set point, while the system utilizing the Linear Quadratic Tracker (LQT) method fails to achieve the expected target. This demonstrates that, for first-order modeling, the LQR controller outperforms the LQT controller in guiding the motor output toward the reference value. In the simulation graph, the green and yellow lines represent the step input signals, while the blue line shows the system's output when controlled by either LQR or LQT. From this visualization, it is evident that LQR provides a more accurate and responsive control output compared to LQT under the same system conditions. The LQR-controlled system reaches the target set point more quickly and with less overshoot, while the output controlled by LQT exhibits larger deviations from the reference. This performance disparity highlights the superior tracking capability of LQR, making it more effective in achieving better accuracy in the motor's response. When noise is introduced into the system, both LQR and LQT controllers still manage to drive the motor output toward the set point, but the presence of noise introduces instability in the system. The yellow line in the simulation indicates the step input signal, while the blue line shows the noisy system response. Despite the added disturbance, the LQR-

controlled system exhibits more fluctuation and its output starts to follow the noise pattern, resulting in a non-linear and unstable response. On the other hand, the LQR system shows some improvement in handling noise but still struggles to maintain stability as effectively as LQR did under ideal conditions. These findings suggest that while both control strategies are effective under ideal, undisturbed conditions, additional modifications or robust control techniques are required to enhance their performance in real-world environments with disturbances. This underlines the need for further research into optimization methods that can maintain system stability and accuracy even under non-ideal conditions [11] [14] [16] [18].

3.2. Simulation Result of DC Motor with noise

Table 3. Simulation result LQR and LQT with noise

| Method | Scope | With Noise | |
|--------|---|------------|------------|
| | | Set Point | Undershoot |
| LQR |  | 10 | - |
| LQT |  | 1 | - |

Based on the results displayed on the scope, it can be observed that the outputs of both the LQR and LQT first-order DC motor systems under noise conditions successfully reach the desired set point. However, the presence of added noise in the input signal causes the resulting graph to appear irregular and less stable. In the simulation output, the yellow line represents the step input signal, while the blue line illustrates the system response from the LQR and LQT controllers applied to the noisy first-order DC motor. The distorted and fluctuating output pattern clearly indicates the

impact of noise on the system's performance, despite the controllers' ability to direct the output toward the target value [8] [9] [12].

4. Conclusion

Based on the experimental results conducted on the DC Motor IG-42CRGM, it was found that the implementation of the LQR (Linear Quadratic Regulator) method significantly improves the motor's output response. This is evident from the behavior of the Maxon EC-I 40 70 Watt DC motor, where the system was able to reach the desired set point in a relatively short time when LQR was applied. In contrast, without the use of the LQR controller, the motor's response deviated considerably from the set point, and it took a much longer time to achieve steady-state conditions. These findings support the theoretical foundation that LQR, as an optimization method, enhances the dynamic performance of DC motors, improving both response time and tracking accuracy. However, when noise was introduced into the system, the LQR controller was unable to maintain the quality of the motor response. The motor's output began to mimic the characteristics of the noise signal, resulting in a non-linear and unstable response. This indicates that while LQR is effective in clean conditions, its robustness may be limited in environments with significant disturbances.

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